Auction Size and Collusion

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First Draft - March 2004

Abstract

This paper analyzes the consequences of the size of an auction, in terms of the volume of product and the time lenght covered by the auctioned contract, on the incentive bidders have to collude. The paper shows that the recommendation to increase as much as possible the auction size in order to prevent bidders from colluding, found in several policy papers, is valid only in some exceptional circumstances. In many cases increasing the auction size may induce bidders to form a cartel that would not have been existed otherwise.

Keywords: Auction, auction design, procurement, collusion

JEL Classification D44 (auctions) H57 (Procurement) L41 (Horizontal Anticompetitive Practices)

1 Introduction

Procurement contracts are often awarded through competitive bidding or tendering processes (in short auctions) in order to guarantee that products are purchased at the lowest feasible price. However, collusion among bidders may impede this outcome and significantly reduce consumer and social welfare. Bid rigging is a widespread phenomenon. In many cases it occurs through explicit and illegal agreements. Their frequecy and pervasiviness are well documented in OECD (1999). Collusion in auctions has attracted large attention from legislators, policymakers, prosecutors, antitrust agencies, courts, laymen and ultimately economists. Economists have a long history in studying both collusion and auctions. For a long time research on these two issues has followed almost parallel paths as they seemed to address rather different questions. Auction theory (summarized in Klemperer, 1999, 2000) investigates the properties of several auction forms assuming fixed entry and lack of collusion. Klemperer (2003) has pointed out that the received theory of auctions has little value in practical auction design. IO theory (summarized in Tirole 1988 and Martin, 1993) investigates collusion without spelling the details of how contracts are

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awarded to sellers. However, starting from the end of 1980's collusion in auctions has become the subject matter of a large and growing theoretical and empirical literature. This literature has pursued several objectives: describing how bid rigging works and their efficiency properties (Graham and Marshall, 1987; Mailath and Zemsky, 1991; McAfee and McMillan, 1992; Brusco and Lopomo, 1999; Pesendorfer, 2000); providing empirical tests to detect collusion in auctions (Zona, 1986; Porter and Zona, 1993, 1997; Baldwin et al., 1997; Bajari and Ye, 2001); and finding optimal auction design to foster competition and discourage collusion (Robinson, 1985; Hendricks and Porter, 1989; McAfee and McMillan, 1987; Thomas, 2001; Klemperer, 2002). The latter group of papers examine the effects on collusion of both the auction form (e.g. ascending versus sealed bid first-price auctions) and some specific auctions rules, such as information disclosure about bidders and bids, allocation rules, reserve price, permissible bids. With the only exception of Thomas (2001) these contributions describe static games as authors assume that single or multiple objects are auctioned in a single auction. However, they all (informally) rely on supergames to explain how bidders overcome the enforcement problem faced by any collusive scheme. Finally, Klemperer (2002) advocates an effective antitrust enforcement to prevent explicit bid rigging.

This paper aims at contributing to the discussion of optimal auction design against collusion, by tackling the issue of the impact of the auction size on the risk of explicit collusion, where the auction size is given by the volume of product and the time lenght covered by the auctioned contract. The subject has been addressed in passing by Klemperer (2002) who proposes to aggregate lots into larger packages to make it harder for bidders to divide the spoils. Similar recommendations can be found in policy documents prepared by public agencies or their staff. For instance, according to OECD (1999, p. 22) "Reducing the number of opportunities in which these firms meet, [...] may reduce the opportunities for punishment and therefore may facilitate competition. This might be achieved, for example, by holding fewer, larger auctions, such as auctions for the right to provide certain services over the next five or ten years. If the period of time is long enough, the individual firms need not fear retaliation in future for undercutting the cartel price today".[1] This suggestion is based on solid results of the economic theory of collusion. Since the seminal paper of Stigler (1964) economists argue that a collusive scheme must solve two problems. The first is agreeing about the terms of coordination. The second is to enforce the concerted action. Klemperer argument posits that a larger auction makes the first problem more severe. This assertion has an intuitive explanation that does not

[1] "Another defensive tactic available to agencies is to combine orders. The existence of a large number of contract opportunities facilitates collusion among sellers. When buyers are numerous, and each purchases only a small amount, sellers have less incentive to grant price cuts. Consolidation of purchases tends to increase the value of winning the bid. A firm, even if part of a conspiracy, may be tempted to cheat and take the prize". This statement can be found, in the same exact wording, in a document by the Irish Competition Authority (undated) and the US DoJ Antitrust Division (undated). Similar statements can be found in Chan, Laplagne and Appels (2003) who suggest to "sell several items in one auction instead of a series of auctions".

require a formal model. The other argument in favor of larger auctions rests on a formal proposition according to which a collusive equilibrium is more difficult to sustain if cheating on the cartel is more profitable and if punishment of defectors is delayed (see Jacquemin and Slade, 1989 or Motta, 2004). By increasing the size of a repeated auction both results are attained. A deviator wins a larger contract which entails larger profits, and if the size of demand is not affected by the size of the auction, auctions take place less frequently, deferring the opportunity for retaliation against the cheater. If the size of the auction does not affect the number of independent bidders and if we consider only *tacit* collusion, these two propositions, that point to the same policy recommendation, are likely to be all we can say. However, the decision to form an *explicit* cartel does not depend only on how difficult it is to find a common understanding and enforce the collective decision. In the present paper I consider a more general condition which refers to the incentive firms have to attempt an illegal collusive scheme.[2] This incentive stems from the expected gain they obtain if they try to form a cartel. While hampering the stability of a cartel reduces the probability of a successful collusion, the expected gain of a cartel may as well rise if the adopted rule determines larger collusive profits. This in turn may induce firms to collude because the expected gain exceeds the expected loss stemming from an antitrust conviction and therefore lead to the formation of a cartel that would not have been formed otherwise. Moreover, increasing the auction size may affect entry conditions and change the number of independent bidders. If this happens the favorable relationship between the auction size, on one side, and the difficulty to reach an agreement and the instability of a collusive equilibrium, on the other side, may not be valid anymore.

Decisions about auction size are driven primarily by efficiency reasons as they depend on scale economies, transaction costs, inventory costs and the like. An optimal decision on auction size should take into account these factors and the impact on collusion at the same time as the welfare consequences of efficiency and collusion could be traded off. However, for analytical purposes I restrict attention to the impact on collusion of several auction sizes, all equally efficient. This condition defines the set of "feasible" auctions. The paper shows that the policy recommendation to increase the auction size as much as possible (i.e. as far as efficiency considerations do not overturn its welfare properties) may be misleading. The optimal auction size, as fas as collusion is concerned, is the largest among the feasible (efficient) auctions only in some exceptional circumstances. In a large number of cases increasing the auction size beyond a certain level may persuade bidders that looking for an explicit collusive mechanism is worthwile.

Although the paper deals explicitly with procurement auctions, its findings apply also to selling auctions with an appropriate interpretation of the relevant variables.

The paper is organized as follows. Section 2 describes the model. Section 3 presents some neat results that can be proved analiticallly, by assuming

[2] This approach is followed by McCutcheon (1997).

quite strict conditions. Although the conditions imposed are rarely satisfied, this analysis helps to understand the relationship between auction size and the incentive to collude based on some features of an auction which depend on its size. As general statements are impossible without adding more structure to the model, section 4 discusses the results of a numerical simulation run to show the interaction among the auction features affected by the choice of its size. Some practical implications for auction design are summarized in section 5. Section 6 concludes. Two appendixes contain the proofs of the propositions stated in section 3 and the simulation discussed in section 4.

2 The model

To study the relationship between auction size and the risk of collusion, I first define a "unitary" auction in which a contract for the provision of the smallest efficient volume of product is awarded to the winner of the auction. Demand over time is satisfied by repeating the unitary auction with the time lag required for the consumption of the volume of product provided in the unitary auction. The size of the auction is represented by the number of periods covered by the auctioned contract, where each period corresponds to the period covered by a unitary auction. Hence, saying that an auction is of size s means that the winner is awarded a contract whereby it supplies the quantity that would be required by s unitary auctions repeated over s periods. This means that s can take only integer values.[3] The mimimum feasible auction is 1 whereas $\overline{s}$ denotes the maximum feasible auction size.

The contract auctioned in an auction of size s can take two forms. The first prescribes that delivery and payment follow the same timing as s independent unitary auctions. Hence the winning firm delivers the product and receives its payment at the beginning of each of the s periods. In the second form exchange and payment take place in the first period and no transactions occur fos $s-1$ periods until the subsequent auction.[4] I call the first instance a "timing invariant contract" as increasing the size of the auction does not affect the timing of the material exchange.

Let n the number of independent firms that can bid in the unitary auction. The number of independent firms in an auction of size s is a non-increasing function of s, $n_s = n(s)$. A negative relationship between the size of the auctioned contract and the number of independent bidders may exist for several reasons. First, firms may be capacity constrained. If the contract is not time invariant, this may force some firms to merge, to form consortia to participate in the auc-

[3] This restriction does not have implications for the propositions stated in section 3 as they concern only cases where optimality is reached at extreme values. Moreover, the assumption reflects what happens in reality as the provision of the auctioned goods or services normally follow some time patterns that would be inefficient to partition. For instance, milk for schools is provided on monthly or annualy basis. Both cases could be accounted for in the model by a unitary auction for a monthly contract.

[4] Of course intermediate cases are possible but this two polar forms suffice to prove the main results of the paper.

tion or to forego the auction altogether. Second, even if capacity constraints do not bind (or if the contract is time invariant) firms may be unwilling to make commitments for long periods and may decide to hedge against risk by pooling their activity with other firms. Third, the value of the guarantee requested to participate in an auction is normally a proportion of the value of the auctioned contract. An imperfect credit market may prevent some firms to obtain the required higher guarantee for a larger auction. Also in this case firms may form temporary group to overcome this problem.[5] If n_s firms compete their expected competitive profits, in each repetition, are zero. If they collude, in each period they obtain collusive profits that individually equal to π/n_s.[6]

The variable δ denotes the discount factor with which firms discount profits to be obtained one period later in the unitary auction. Let δ^* be the critical discount factor such that if $\delta \geq \delta^*$, firms can sustain a collusive outcome with trigger strategies, which incorporates optimal punishments given the assumption on profits. The economic literature studies the impact of several market features on collusion by finding their relationship with the critical discount factor.[7] Results are normally framed in propositions that state that a given modification of some market characteristic increases the critical discount factor, and therefore hinders collusion, or decreases the critical discount factor, and therefore facilitates collusion. Some authors verbally frames their results in terms of probability by saying that some factors increases (or decreases) the likelihood of collusion. Crispier propositions are not possible because, unless δ^* takes extreme values, the market game always possesses infinite equilibria, provided that the actual discount factor is large enough. In the present paper, I translate the approach followed in the economic literature in terms of probability by attaching a probability that depends on δ^* to the event "successful cartel". Denote with p_t the probability that the collusive scheme is successful in the auction that takes place in period t. I assume that for the first auction this probability is a non-increasing function of δ^*, $p_1 = p(\delta^*)$.[8] For subsequent auctions we can make two alternative assumptions. The first possibility is that the probability of a successful cartel is the same in each repetition independently of the outcome of previous auctions, i.e. $p_t = p(\delta^*)$ for any t. If this holds, we

[5] One may argue that adding more objects to an auction may attract bidders that would not have participated otherwise becuase the participation costs were higher than expected returns. This possibility is excluded in the model by the asssumption that auctions of different size are all equally efficient. Indeed, the possible objection rests on the existence of transaction (participation) costs that do not vary (proportionally) with the value at stake. The efficiency assumption in the model does not negate validity of this argument but serves the only purpose of disentangling efficiency and collusion as far as possible.

[6] A symmetrical repartition of collusive profits is not a necessary assumption, but simplifies the exposition.

[7] See Bernheim and Whinston (1990) and Spagnolo (1999) (multi-market contact); Rotember and Saloner (1986) (evolution of demand); Spagnolo (2000) (managerial compensation schemes), or Motta (2004, ch, 4) for a survey and a general discussion.

[8] This generic formulation may incorporate also the intuition behind the suggestion made by Klemperer (2002) and cited in the Introduction. As we shall see, δ^* is an increasing function of s. Therefore, if $p(\delta^*)$ is negatively sloped this may reflect both the coordination and the enforcement problems of an explicit cartel.

say that p_t is stationary. The opposite assumption is that if the cartel breaks down in an auction the probability of a successful ring in subsequent auctions is nil.[9] Therefore:

$$p_t = \begin{cases} [p(\delta^*)]^t & \text{if firms colluded in precedent periods} \\ 0 & \text{otherwise} \end{cases}.$$

It must be understood that I do not model the uncertain factors that give rise to this probability. Moreover as δ^* is only a proxy of how diffifult is to enforce a cartel, p_t does not enter in its calculation.

Firms decide to form a bidding ring if the expected profits from the collusive scheme excedes the expected sanction stemming from an antitrust conviction. I assume that the incentive to collude increases if the collusive profits increase. Several conditions can validate this assumption. The simplest one is that auction size (and the present value of the flow of collusive profits) does not modify the probability of an antitrust investigation and the fine and other losses imposed on firms in case of conviction. However, the assumption is still valid if the expected loss increases with the auction size, but less than proportionally than collusive profits. This assumption is routinely, though implicitly, made in all models of collusion (and all papers cited in the Introduction are no exceptions) where firms collusive goal is to maximize joint profits.[10] Hence, the optimal auction size, as far as collusion is concerned, is the one that minimizes the expected profits of forming a cartel.

3 Results

In this section I analyze how the auction size alters the incentive to collude. I first consider the case with a fixed number of bidders. If $n_s = n$ for any s, a clear positive relationship exists between δ^* and s whether the contract is time invariant or not. In the first case if a firm respects the collusive agreement it obtains:

$$\sum_{t=0}^{\infty} \frac{\pi}{n} \delta^t = \frac{\pi}{n(1-\delta)},$$

whereas, by defecting it gets:

$$\sum_{t=0}^{s-1} \pi \delta^t.$$

[9] Also in this case there may be intermediate situations but for the sake of simplicity we limit our analysis to these two extreme cases. Moreover the hypothesis of a stationary probability seems at odds with the trigger strategies used to compute δ^*. However, I do not exlude this possibility because it can be considered a limit case and because δ^* is only a proxy of how difficult is to collude but does not imply that firms must use trigger strategies.

[10] For a different and extremely interesting approach see Harrington (2003).

Cheating is not a rational choice if and only if:

$$\delta \geq \left(\frac{n-1}{n}\right)^{\frac{1}{s}} = \delta^*.$$

If the contract is not time invariant, colluding entails the following individual profits:

$$\sum_{t=0}^{\infty} \frac{s\pi}{n} \delta^{st} = \frac{s\pi}{n\left(1-\delta^s\right)},$$

while, by deviating a firm can obtain:

$$s\pi.$$

Also in this case respecting the collusive scheme is rational if and only if:

$$\delta \geq \left(\frac{n-1}{n}\right)^{\frac{1}{s}} = \delta^*.$$

Hence, δ^* is an increasing function of s. Having established this result, the first three propositions describe the impact of s on the incentive to collude (i.e. on the expected flow of collusive profits) when the number of independent bidder is constant (all proofs are in Appendix A).

Proposition 1 *If the number of indipendent bidders is constant, the auctioned contract is time invariant, p_t is stationary and $p(\delta^*)$ is a decreasing function of δ^*, then the optimal auction size is $\overline{s}$.*

The first proposition formalizes the intuition behind the policy recommendation cited in the Introduction. A larger auction reduces the frequency of bidders interactions and delays the punishment firms can impose on deviators. This hampers the stability of collusion and, if p_t is stationary and the other contractual and market conditions are not affected by the choice of the auction size, diminishes the incentive to collude, by reducing the probability of a successful collusion.

The following results contains the hypothesis that the probability of a successful cartel is constant in order to single out the effects of the auction size on the other determinants of the incentive to collude.

Proposition 2 *If the number of independent bidders is constant, the probability of a successful cartel is constant, p_t is stationary and the auctioned contract is not time invariant, then the optimal auction size is 1.*

Proposition 2 shows that if by increasing the auction size, the material exchange of goods (and payment) is also affected, bidders have an higher incentive to collude (ceteris paribus) because they can reap earlier some of the collusive profits that would be obtained in the future and therefore valued less.

Proposition 3 *If the number of independent bidders is constant, the probability of a successful cartel is constant, the auctioned contract is time invariant and p_t is non-stationary, then the optimal auction size is 1.*

The intuition for proposition 3 is also very simple. If the probability of a successful cartel decays over time, concentrating a larger volume of product in earlier contracts increases the expected flow of collusive profits as these profits would be obtained with higher probabilities.

It must be noted, that propositions 2 and 3 are based on two separate effects of a modification of the auction size. Therefore these results may be combined by saying that the optimal aucion size is 1 if the contract is not time invariant and/or p_t is stationary, provided that the other assumptions in the two propositions hold.

Now, I investigate how a modification of the auction size affects the incentive to collude via a modification of the number of independent firms that can bid in the auction. General results cannot be established as also the sign of the change of the critical discount factor is not clear cut. Indeed, if s increases and, as a consequences of this, n_s decreases, changing the size of the auction will affect the critical discount, $\delta^* = \left(\frac{n_s-1}{n_s}\right)^{\frac{1}{s}}$, in two opposite ways. It decreases because n_s decreases, and increases because s increases. Which of these two effects prevails cannot be said *a priori*. A general statement is obtainable only if we assume that the probability of a successful cartel does not depend on δ^*.

Proposition 4 *If the probability of a successful cartel is constant, and the number of independent firms is a decreasing function of s, then the optimal auction size is 1.*

Proposition 4 rests on the assumption that what matters are the individual incentives to collude and not the joint profits gained by the collusive ring.[11] However this result is still valid if we consider the collective incentive to form a cartel and if we assume that the costs of reaching an agreement and policing its implementation are increasing in the number of collusive bidders.

Finally, a comparison between time invariant contracts and contract that are not time invariant leads to a result of general validity as stated in the following proposition.

Proposition 5 *For any $s > 1$, the incentive to collude is always higher if the contract is not time invariant.*

Proposition 5 states that time invariant contract dominates not time invariant contracts, as far as the objective of fighting collusion is concerned, whatever

[11] Indeed, this seems to be the only sensible assumption. Although *how* to collude is a collective decision, *whether* to collude in the first place is an individual decision that must be individually incentive compatible (see McAfee and McMillan, 1992, among others).

the other conditions are. Its validity depends on the fact that in any case the present value of of the flow of collusive profits is higher if payments are concentrated in earlier periods.

4 A numerical simulation

If we take into account all the possible effects of varying s on the incentive to collude the model becomes analitically intractable. Therefore in Appendix B I run a numerical simulation to describe some possible patterns. The simulation shows that the relationship between the auction size and the incentive to collude can take any form. It depends primarily on two features of $p(\delta^*)$. These are: its maximum value and the rapidity with wich it diminishes as s (and δ^*) increases. If p_t is stationay the second characteristic is fundamental. Indeed, the stronger is the impact of s on $p(\delta^*)$ the larger is the optimal auction size. If p_t is not stationary, also the slope of the relationship between the auction size and the incentive to collude depends both on the maximum value of $p(\delta^*)$ and on the rapidity of its decline. Finally, if the number of indpendent bidders is a (possibly non monotone) decreasing function of s the optimal auction size is often the largest among those compatible with the highest number of bidders if p_t is stationary and the contract is time invariant. However, if these conditions do not hold it may be preferable to have a smaller auction or even a lower number of bidders but a larger auction.

5 Some practical implications

Even if the results of the model are quite vague in the general case, I believe that some practical implications can be drawn from the model presented in this paper. The first indication is that, independently of the size of auctioned contract, the timing of the actual transaction should be broken up as much as possible, given efficiency limits. This suggestion stems from proposition 5. The second indication is that a procurer should consider carefully the impact of the auction size on the number of independent bidders. Even if there is not a clear-cut relationship between the auction size and the incentive to collude when the former affects (among the other things) entry conditions, the simulation shows that the incentive to collude may increase significantly as the number of bidders decreases. Moreover, one should consider that also the critical discount factor may be reduced by limiting the participants in the auction and that, therefore, this may improve also the chances of some form of tacit collusion. The general recommendation is not to apply the suggestions provided by OECD or other agencies, without questioning their actual and practical validity.

6 Conclusions

I have analyzed the consequences of the size of an auction, in terms of the volume of product and the time lenght covered by the auctioned contract, on the incentive bidders have to collude explicitly. The paper shows that the recommendation to increase as much as possible the auction size in order to prevent bidders from colluding is valid only in some exceptional circumstances. In many cases increasing the auction size may induce bidders to form a cartel that would not have been existed otherwise.

Appendix A - Proofs

Proposition 1

Proof. Suppose $n_s = n$ for any s, the critical discount factor is $\delta^* = \left(\frac{n-1}{n}\right)^{\frac{1}{s}}$ and therefore is an increasing function of s. If the contract is time invariant, the present value of collusive profits is

$$\sum_{t=1}^{\infty} p_t \left(\sum_{r=1}^{s} \frac{\pi}{n} \delta^{st-r} \right). \ (1)$$

Since, $p_t = p(\delta^*)$ for any t, equation (1) becomes:

$$p(\delta^*) \sum_{t=1}^{\infty} \sum_{r=1}^{s} \frac{\pi}{n} \delta^{st-r} = p(\delta^*) \frac{\pi}{n} \sum_{t=1}^{\infty} \delta^{t-1} \quad (2)$$

As $p(\delta^*)$ is a decreasing function of δ^* and δ^* an increasing function of s, equation (2) is a monotonically decreasing function of s that reaches its minimum at $s = \overline{s}$. ■

Proposition 2

Proof. The present value of the flow of collusive profits if the contract is not time invariant is:

$$\sum_{t=1}^{\infty} p_t \frac{s\pi}{n_s} \delta^{st-s} \ (3).$$

If the number of indenpendent bidders and the probability of a successful cartel are both constant and p_t is stationary, equation (3) becomes:

$$p \frac{s\pi}{n} \sum_{t=0}^{\infty} \delta^{st} = p \frac{s\pi}{n(1-\delta^s)} = \frac{sa}{1-\delta^s},$$

where $a = p\frac{\pi}{n} > 0$. Now consider how the incentive to collude changes if the auction size is increased from s to $s+1$. We have:

$$\frac{(s+1)a}{1-\delta^{s+1}} - \frac{sa}{1-\delta^s} = a \frac{1-\delta^s(1+s(\delta-1))}{(1-\delta^{s+1})(1-\delta^s)} > 0 \text{ if } \delta^s(1+s(\delta-1)) < 1.$$

Since $\delta < 1$, this inequality always holds as $\delta^s < 1$ and $1 + s(\delta - 1) < 1$. Hence, by increasing the size of the auction the incentive to collude increases, therefore the optimal auction size is 1. ■

Proposition 3

Proof. The present value of the flow of collusive profits is:

$$\sum_{t=1}^{\infty} p_t \left(\sum_{r=1}^{s} \frac{\pi}{n_s} \delta^{st-r} \right) \ (4).$$

Consider two auctions of size s and $s+1$. Given the asumptions in the proposition they entail the following flow of profits:

$$b = \frac{\pi}{n}\left(p\left(1+\delta+...+\delta^{s}\right)+p^{2}\left(\delta^{s+1}+...+\delta^{2s+1}\right)+p^{3}\left(\delta^{2s+2}+...+\delta^{3s+2}\right)+...\right)$$

$$c = \frac{\pi}{n}\left(p\left(1+\delta+...+\delta^{s-1}\right)+p^{2}\left(\delta^{s}+...+\delta^{2s-1}\right)+p^{3}\left(\delta^{2s}+...+\delta^{3s-1}\right)+...\right).$$

Subtracting c to b, we obtain a sequence in which all terms have the following structure:

$$\frac{\pi}{n}\delta^{t}\left(p^{\alpha_t}-p^{\beta_t}\right)$$

with $\alpha_t < \beta_t$ for any t. Since $p < 1$, all the sequence terms are positive, which implies that their summation is also positive. As the incentive to collude is an increasing function of s, the optimal auction size is 1. ■

Proposition 4
Proof. The proof is trivial as if $n_s > n_{s+1}$ then the individual collusive profit are higher if the auction size increas from s to $s+1$. In a game with a time invariant contract and p_t stationary this suffices to increase the incentive to collude. This result holds a fortiori if p_t is non stationary and/or the contract is not time invariant, as established in propositions 2 and 3. ■

Proposition 5
Proof. If p_t is stationary, the flow of collusive profits with a not time invariant contract is: ■

$$\frac{s\pi}{n}p\sum_{i=0}^{\infty}\delta^{is},\ (5)$$

whereas if the contract is time invariant it is:

$$\frac{\pi}{n}p\sum_{i=0}^{\infty}\delta^{i}\ (6).$$

By subtracting (6) to (5) we get:

$$\frac{\pi}{n}p\left(\frac{s}{1-\delta^{s}}-\frac{1}{1-\delta}\right) > 0 \text{ for any } s > 1.$$

If p_t is not stationary, the flow of collusive profits is with a not time invariant contract:

$$\frac{s\pi}{n}\sum_{i=1}^{\infty}p^{i}\delta^{s(i-1)},\ (7)$$

and with a time invariant contract:

$$\frac{\pi}{n}\sum_{i=1}^{\infty}p^{i}\sum_{r=1}^{s}\delta^{si-r}\ (8).$$

By subtracting (8) to (7) we get:

$$\frac{\pi}{n}\left(\sum_{i=1}^{\infty}p^{i}\left((s-1)\,\delta^{s(i-1)}-\sum_{r=1}^{s-1}\delta^{s(i-1)+r}\right)\right).$$

Since

$$\delta^{s(i-1)+r}<\delta^{s(i-1)}\text{ for any }r>1,$$

we have that

$$\begin{aligned}\left((s-1)\,\delta^{s(i-1)}-\sum_{r=1}^{s-1}\delta^{s(i-1)+r}\right) &> \\ \left((s-1)\,\delta^{s(i-1)}-(s-1)\,\delta^{s(i-1)}\right) &= 0.\end{aligned}$$

Appendix B - Model of the numerical simulation

The probability of a successful collusion is described by the following function:

$$p(\delta^*, \gamma, \theta) = \gamma (1 - \delta^*)^{\theta},$$

where $\gamma \in (0, 1)$ represents its maximum value, and $\theta \geq 0$ represents the rapidity with wich it diminishes as δ^* increases. If $\theta = 0$, p is constant. In all simulations I pose $s = 1, 2, ..., 20$, $\pi = 1$, $\delta = .99$, and in those simulations where the number of bidders is independent of s, $n = 2$. In the four cases where n is a decreasing funtion of s, the following function holds:

s	1	2	3	4	5	6	7	8	9	10	11
n	6	6	6	6	5	5	5	5	5	4	4

s	12	13	14	15	16	17	18	19	20
n	4	4	3	3	3	3	3	2	2

The number of repetitions of the unitary auction is 1000. This allows to approximate the present value of the flow of profits in the infinite repetition case also when s takes the highest value.

The following tables summarizes the hypotheses formulated in each of the 11 simulations, plus those of the four propositions of section 3.

	A	B	C	D	E	F	G (prop.1)
n constant	yes	yes	yes	yes	yes	yes	yes
p stationary	yes	yes	no	no	no	no	yes
γ	.5	.5	.9	.6	.6	.9	.5
θ	.05	.03	.9	.9	1	.9	.05
time invariant	no	no	no	no	no	yes	yes

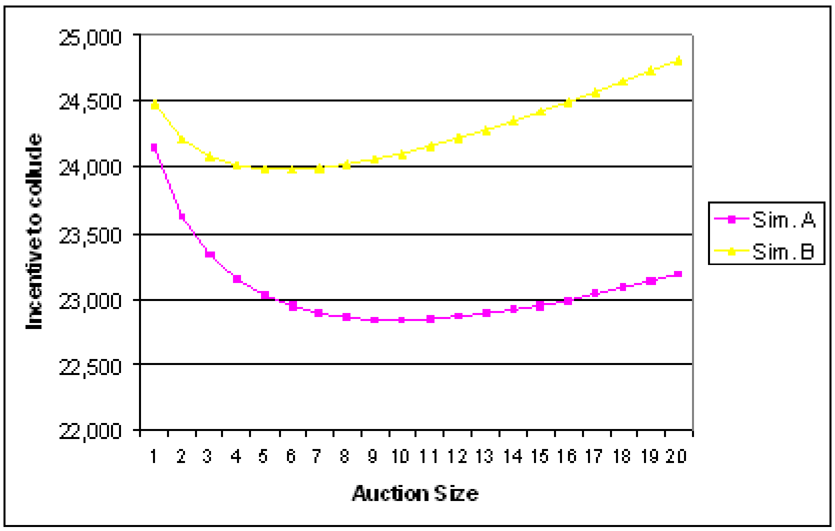
	H	I	L	M	prop 2	prop 3	prop 4
n constant	no	no	no	no	yes	yes	no
p stationary	yes	yes	no	no	yes	no	yes/no
γ	.9	.9	.9	.9	$0 \leq \gamma \leq 1$	$0 \leq \gamma \leq 1$	$0 \leq \gamma \leq 1$
θ	.9	.9	.9	.9	0	0	0
time invariant	no	no	no	no	no	yes	yes/no

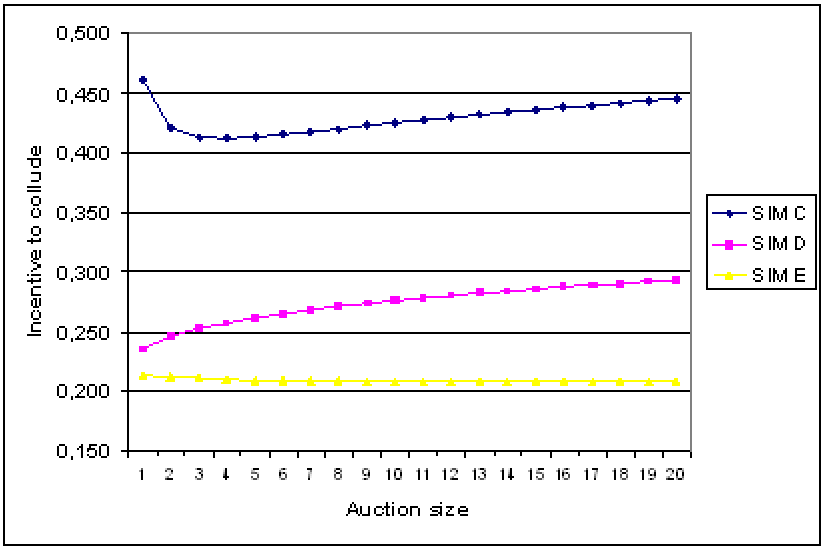
The following tables, where the minimum value of the incentive to collude is in bold, summarize the results of the simulation.

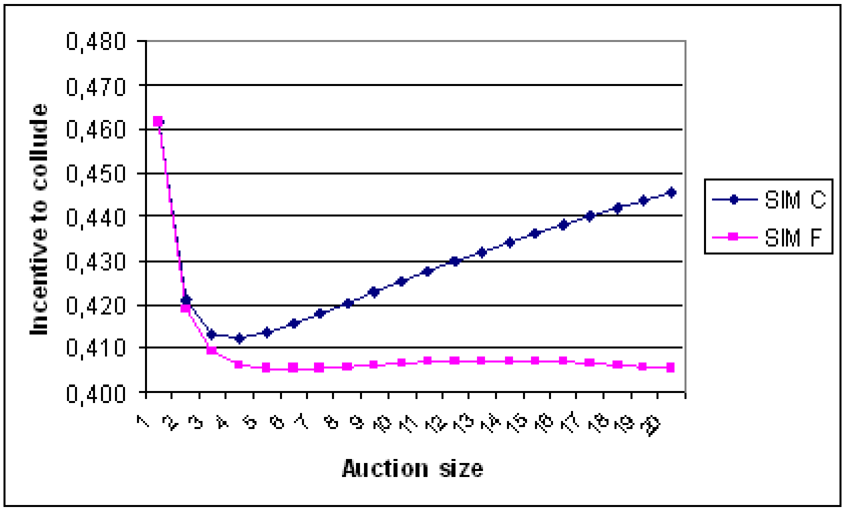
s	A	B	C	D	E	F
1	24.147	24.484	.462	**.236**	.2134	.4615
2	23.628	24.216	.421	.247	.2123	.4189
3	23.334	24.083	.413	.253	.2110	.4092
4	23.149	24.016	**.412**	.258	.2102	.4062
5	23.026	23.987	.414	.262	.2097	.4054
6	22.944	**23.983**	.416	.266	.2093	**.4053**
7	22.890	23.997	.418	.269	.2090	.4056
8	22.858	24.025	.420	.271	.2088	.4060
9	22.842	24.062	.423	.274	.2086	.4063
10	**22.839**	24.108	.425	.276	.2085	.4066
11	22.846	24.160	.428	.279	.2084	.4069
12	22.863	24.218	.430	.281	.2083	.4070
13	22.886	24.281	.432	.283	.2082	.4071
14	22.916	24.348	.434	.284	.2081	.4071
15	22.951	24.418	.436	.286	.2081	.4070
16	22.991	24.491	.438	.288	.2080	.4069
17	23.035	24.568	.440	.289	.2080	.4067
18	23.083	24.646	.442	.291	.2079	.4064
19	23.134	24.727	.444	.292	.2079	.4060
20	23.189	24.810	.446	.293	**.2079**	.4056

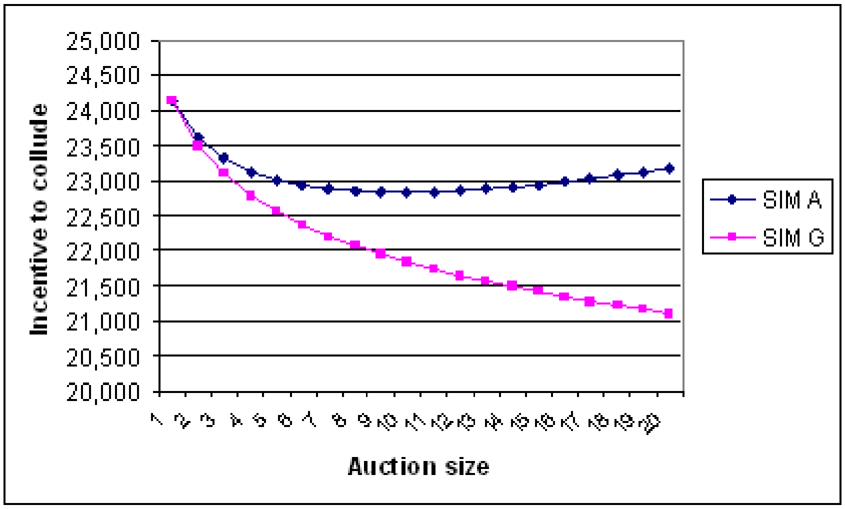
s	G	H	I	L	M
1	24.147	2.990	7.619	**.036**	**.036**
2	23.510	1.676	7.376	.037	.037
3	23.102	1.186	7.233	.038	.037
4	22.804	.926	**7.133**	.039	.038
5	22.570	1.096	8.550	.057	.056
6	22.378	.938	8.474	.057	.056
7	22.215	.823	8.410	.058	.056
8	22.074	.734	8.355	.059	.057
9	21.949	**.665**	8.307	.059	.057
10	21.838	.953	10.460	.094	.090
11	21.738	.880	10.411	.095	.090
12	21.646	.818	10.366	.096	.091
13	21.562	.766	10.325	.096	.091
14	21.484	1.304	13.951	.177	.166
15	21.412	1.232	13.904	.178	.166
16	21.345	1.169	13.859	.179	.166
17	21.281	1.113	13.818	.180	.166
18	21.222	1.063	13.779	.180	.166
19	21.166	2.458	21.166	.444	.406
20	**21.112**	2.360	21.112	.446	.406

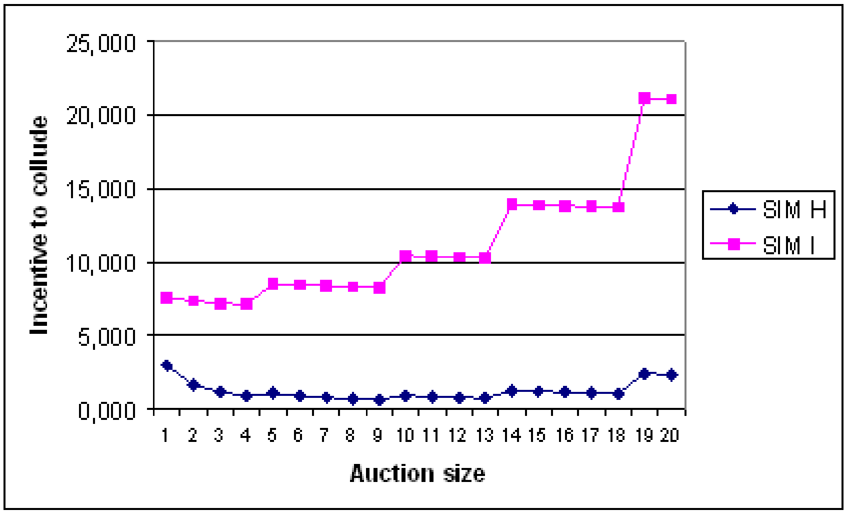
The following figures depict the relationships between the auction size and the incentive to collude, provide a comparison between similar situation and help to read the tables.

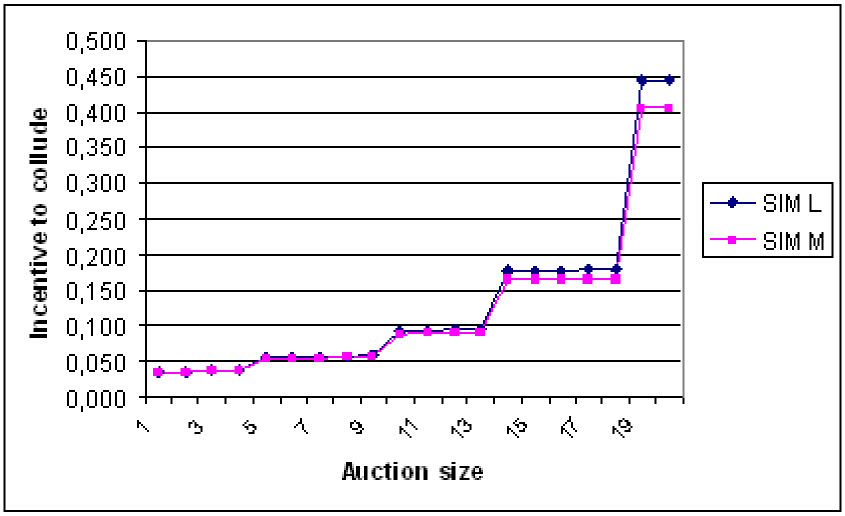



0,500
0,450
0,400
0,350
0,300
0,250
0,200
0,150
Incentive to collude
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
Auction size
SIM C
SIM D
SIM E


0,480
0,470
0,460
0,450
0,440
0,430
0,420
0,410
0,400
Incentive to collude
Auction size
SIM C
SIM F


25,000
24,500
24,000
23,500
23,000
22,500
22,000
21,500
21,000
20,500
20,000
Incentive to collude
Auction size
SIM A
SIM G


25,000
20,000
15,000
10,000
5,000
0,000
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
Incentive to collude
Auction size
SIM H
SIM I



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