Tying in Two-Sided Markets with Multi-Homing

by

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Abstract

This paper analyzes the effects of tying arrangements on market competition and social welfare in two-sided markets when economic agents can engage in multi-homing; that is, they can participate in multiple platforms in order to reap maximal network benefits. The model shows that tying induces more consumers to multi-home and makes platform-specific exclusive contents available to more consumers, which is also beneficial to content providers. As a result, tying can be welfare-enhancing if multi-homing is allowed, even in cases where its welfare impacts are negative in the absence of multi-homing. The analysis thus can have important implications for recent antitrust cases in industries where multi-homing is prevalent.

JEL Classification: L1, L4.
Keywords: tying, two-sided markets, (indirect) network effects, multi-homing.

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I. Introduction

This paper analyzes the effects of tying arrangements on market competition and social welfare in two-sided markets when consumers can engage in multi-homing; that is, consumers can participate in multiple platforms (or purchase multiple products) in order to reap maximal network benefits. The paper is partly motivated by the recent antitrust cases concerning Microsoft. In the European case, for instance, it has been alleged that the company’s tying practice of requiring Windows operating system users to accept its Windows Media Player software is anticompetitive and hurts digital media rivals such as RealNetworks.¹ However, multi-homing is common in digital media systems. Many users have more than one media player and many content providers offer contents in more than one format, which counteracts the tendency towards tipping and the lock-in effects in industries with network effects.

To analyze the effects of tying in markets such as digital media, I adopt the framework of platform competition in two-sided markets. The defining characteristics of two-sided markets are indirect network effects or inter-group network externalities that arise through improved opportunities to trade with the other side of the market. In the digital media case, content providers and final consumers constitute the two sides that trade with each other. For instance, as more contents are available in streaming media, the more valuable media player programs become, and vice versa. Other prominent examples of economic importance include auction sites such as eBay and Yahoo, where buyers and sellers interact to consummate a deal, credit card payment systems such as Visa and

¹ On March 24, 2004, the European Union ruled that Microsoft is guilty of abusing the "near-monopoly" of its Windows PC operating system and fined it a record 497 million euros ($613 million). The case is being appealed by Microsoft.
MasterCard where both merchants and consumers need to participate in the same system, video game platforms such as PlayStation, Xbox and GameCube where game developers and consumers constitute the two distinct sides, etc.²

I show that tying induces more consumers to multi-home and makes platform-specific exclusive contents available to more consumers, which is also beneficial to content providers. As a result, tying can be welfare-enhancing if multi-homing is allowed, even in cases where its welfare impacts are negative in the absence of multi-homing. The analysis thus can have important implications for recent antitrust cases and provides a caution in applying the traditional theory of network effects and tipping to markets where multi-homing is prevalent.

The analysis in this paper builds on the burgeoning literature on competition in two-sided markets. More specifically, in two-sided markets the need for all sides of the market to get on board creates a so-called “chicken and egg” problem (Caillaud and Jullien, 2003) in that members of each group are willing to participate in the market only if they expect many members from the other side to participate. The literature on two-sided markets is mainly concerned with the optimal pricing structure to coordinate the demands of distinct groups of customers who need each other in some way.³ A fundamental insight of the literature is that a pricing structure that determines price allocation is as important as the overall price levels. However, they are not concerned with implications of the multi-sided nature of the market for antitrust analysis. In particular, they do not analyze how tying arrangements can affect competition in these markets.

² See Evans (2003) and Rochet and Tirole (2003a) for more examples of multi-sided markets.
³ See Armstrong (forthcoming) and Rochet and Tirole (forthcoming).
More recently, the importance of studying antitrust issues in two-sided markets has been recognized by several authors. Evans (2003) and Wright (2003), for instance, provide a general discussion on antitrust policy in two-sided markets and call for caution in applying the traditional one-sided logic to two-sided markets in the antitrust arena. However, their discussion is mainly informal and does not deal with tying arrangements.

Rochet and Tirole (2003b) and Amelio and Jullien (2006) are notable exceptions in the analysis of tying in two-sided markets. Rochet and Tirole provide an economic analysis of the tying practice initiated by payments card associations Visa and MasterCard in which merchants who accept their credit cards were forced also to accept their debit cards. They show that in the absence of tying, the interchange fee between the merchant’s and the cardholder’s banks on debit is too low and tends to be too high on credit compared to the social optimum. Tying is shown to be a mechanism to rebalance the interchange fee structure and raise social welfare. Their model, however, is tailored to analyze the payment card industry and the recent antitrust suit involving Visa and MasterCard. In particular, the analysis focuses on tying by a non-profit association to reflect the status of credit card associations.

Amelio and Jullien (2006) are closest to my paper in that they provide a more general analysis of tying in two-sided markets. They consider a situation in which platforms

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4 This tie-in practice, the so-called “honor-all-cards” rule, has been challenged recently by major merchants, including Walmart, in a class action suit. In the class action suit on behalf of thousands of retailers, the stores argued that Visa and MasterCard unfairly required merchants to accept their debit cards, which required a customer's signature to verify a transaction, to exclude PIN-based on-line debit cards. The case was originally filed in 1996. It was certified as a class action in February of 2000. The trial was set to commence on April 28, 2003, following the defendants' unsuccessful appeals of the class-certification decision and supplementation of summary judgment motions. However, Visa and MasterCard each separately agreed to settle the antitrust lawsuit brought against them. Under the settlements, Visa is set to pay $2.025 billion to merchants over the next 10 years and MasterCard is set to pay $1.025 billion over the same period. They also agreed to drop their "honor all cards" policy, which will allow retailers to accept their credit cards without also accepting debit cards.
would like to set prices below zero on one side of the market to solve the demand coordination problem in two-sided markets, but are constrained to set non-negative prices. In the analysis of Amelio and Jullien, tying can serve as a mechanism to introduce implicit subsidies on one side of the market in order to solve the aforementioned coordination failure in two-sided markets. As a result, tying can raise participation on both sides and can benefit consumers in the case of monopoly platform. In a duopoly context, however, tying also has a strategic effect on competition. They show that the effects of tying on consumer surplus and social welfare depend on the extent of asymmetry in externalities between the two sides. Their paper and mine focus on different aspects of tying and can be viewed as complementary. For instance, they compare the effects of tying across different market structures (monopolistic vs. duopolistic), but they assume single-homing and do not analyze implications of multi-homing in two-sided markets.

This paper is also closely related to the literature on the “leverage theory” of tying. According to the "leverage theory" of tying, a two-product firm with monopoly power in one market can monopolize a second market using the leverage provided by its monopoly power in the first market. Whinston (1990), for instance, shows that if the market structure in the tied good market is oligopolistic and scale economies are present, tying can be an effective and profitable strategy to alter market structure by making continued operation unprofitable for tied good rivals. It is important to keep in mind, however, that in Whinston’s basic model, inducing the exit of the rival firm is essential for the profitability of tying arrangements.\footnote{Whinston (1990) points out that if the heterogeneity of consumer preferences are allowed for the tying good, tying can also serve as a price discriminating device and exclusion of the rival firm is not necessary for the profitability of tying. See also Carbajo et al. (1990).} Thus, if the competitor has already paid the sunk cost of entry and

\footnote{Whinston (1990) points out that if the heterogeneity of consumer preferences are allowed for the tying good, tying can also serve as a price discriminating device and exclusion of the rival firm is not necessary for the profitability of tying. See also Carbajo et al. (1990).}
there is no avoidable fixed cost, tying cannot be a profitable strategy.\textsuperscript{6} Choi and Stefanadis (2001) extend the analysis by investigating implications of tying for innovation incentives. They show that when an incumbent monopolist faces the threat of entry in systems markets consisting of complementary components, tying may make the prospects of successful entry less certain, discouraging rivals from investing and innovating.\textsuperscript{7}

Carlton \& Waldman [2002] are especially worth mentioning in relation to this paper. They investigate how the tying of complementary products can be used to preserve and create monopoly positions. Their analysis focuses on two mechanisms through which tying can be used in an anticompetitive way: entry costs and network externalities. In particular, their model with network externalities shows that the presence of network externalities for the complementary good can result in the strategic use of tying to deter entry into the primary market. The nature of network effects in their paper, however, is direct and thus does not explicitly accounts for the peculiarities of two-sided markets. In addition, none of these papers in the tying literature seriously take into consideration the possibility of multi-homing. Carlton and Waldman (2002), for instance, assume that “if a consumer purchases a tied good consisting of one unit of the monopolist’s primary good and one unit of its complementary good, then the consumer cannot add a unit of the alternative producer’s complementary good to the system (italics added, p. 199).” In other words, either they do not allow the possibility of multi-homing or multi-homing does not arise in equilibrium.

\textsuperscript{6} Carbajo, de Meza, and Seidman (1990) and Chen (1997) provide an alternative theory for strategic bundling in which bundling plays the role of a product-differentiation device. As in this paper, bundling does not require the exit of the rival firm to be profitable. However, bundling is used to segment the market and relax competition.

\textsuperscript{7} In related papers, Choi (1996, 2004) demonstrates that even in the absence of exit by the rival firm, bundling can be a profitable strategy via its long-term effects on competition through innovation.
Doganoglu and Wright (2006a) explicitly consider multi-homing as a way to reap greater network benefits and analyze its implications for price competition. Their focus, however, is on questioning if multi-homing can be a substitute for compatibility. Finally, Carrillo and Tan (2006) analyze the role of multi-homing and complementors in platform competition. In particular, they study the incentives of the platform to integrate with its complementors. In contrast, my focus is on the effects of tying on competition in the presence of multi-homing. Currently, formal economic analysis of tying that explicitly accounts for the possibility of multi-homing is virtually non-existent. The analysis in the paper intends to fill this gap in the literature and can have important implications for recent antitrust cases in industries where multi-homing is prevalent.

The remainder of the paper is organized in the following way. In section II, I set up a basic model of two-sided markets. In section III, I analyze the effects of tying arrangements on competition in two-sided markets in the absence of multi-homing. Section IV considers the possibility of multi-homing in the analysis of tying. Concluding remarks follow.

II. The Basic Model of Two-Sided Markets

In this section, I lay out a very simple model of two-sided markets and derive the market outcome in the absence of tying and single-homing consumers. The model is a modification of the framework developed by Armstrong (forthcoming) and Rochet and Tirole (2003a). The analysis in this section will be used as a benchmark to investigate the effects of tying in two-sided platform markets with single-homing consumers in section III.
In section IV, I will modify the model to account for the possibility of multi-homing on both sides of the market.

The model comprises three classes of agents. There are two distinct types of customer groups that interact with each other and intermediaries who provide platforms to enable these two customer groups to “meet” each other. In the example of streaming multi-media players, the two customer groups can be described as content providers and consumers who download content through the Internet. There are currently three major platform providers: RealNetworks, Microsoft and Apple. In my model, these software/platform providers can be considered the intermediaries who compete in two-sided platform businesses.

Let me assume that there are two intermediaries indexed by $i = A, B$. For concreteness, let me label the two customer groups as content providers and consumers (as in the streaming media industry). The two intermediaries compete for market share within each group. Let $p_i$ and $q_i$ denote intermediary $i$’s charge to content providers and consumers, respectively, where $i = A, B$. The intermediaries’ costs of serving each content provider and final consumer are given by $c$ and $d$, respectively. Finally, the number of content providers and consumers who participate in platform $i$ are denoted by $m_i$ and $n_i$, respectively. As in Armstrong (forthcoming), I consider a situation in which at least one side of the market is characterized by exclusive intermediation. More specifically, I assume that final consumers “single-home,” that is, they participate in only one platform.\(^8\)

\(^8\) The framework is similar to the competitive bottleneck model in Armstrong (forthcoming) that is applied to describe supermarket competition where consumers visit only one supermarket while suppliers typically stock their products on the shelves of several supermarkets.
II.1. Content Providers

I assume that there is free entry into the market for content provision. Content providers are heterogeneous in their fixed cost of creating content, which is denoted as $\theta$. The content providers incur this cost twice if they multi-home, i.e., make their contents available in digital form on both platforms. I normalize the number of potential content providers to 1 and let $F(\theta)$ be the distribution function for $\theta$.

Each content provider gains additional utility (profit) of $\pi$ from each consumer who has access to its content. The profit for content providers who create content on platform $i$ is given by $\pi n_i - p_i - \theta$ when the fixed cost of creating content is $\theta$ and the number of final consumers who participate in platform $i$ is $n_i$. A type-$\theta$ content provider is willing to create content for platform $i$ if $\theta < \pi n_i - p_i$. Thus, the number of content providers on platform $i$ is given by:

$$m_i = F(\pi n_i - p_i) \quad (1)$$

This implies that the larger the number of consumers participating in platform $i$ is, the greater the amount of content that will be provided on that platform.

II.2. Consumers

To analyze the consumers’ choice of platform, I adopt the Hotelling model of product differentiation. I assume that two platforms, $A$ and $B$, are located at the end points of a line with length equal to 1. Consumers, whose size is normalized to 1, are uniformly distributed along the line. Each consumer’s utility of participating in a platform depends on the number of content providers on the same platform. More specifically, the availability of each additional content provider generates additional utility of $b$. Consumers are assumed to
single-home as in the competitive bottleneck model of Armstrong (2007). If a consumer located at point \(x\) participates in platform \(A\), his utility is given by \(b m_A - q_A - tx\) while his utility from participating in platform \(B\) is given by \(b m_B - q_B - t(1-x)\). Assuming that the consumer market is covered, this specification implies that the number of consumers participating in platforms \(A\) and \(B\) are, respectively, given by:

\[
\begin{align*}
n_A &= \frac{1}{2} + \frac{b(m_A - m_B) - (q_A - q_B)}{2t} \\
n_B &= 1 - n_A = \frac{1}{2} + \frac{b(m_B - m_A) - (q_B - q_A)}{2t}
\end{align*}
\]  

(2)

II.3. Platform Competition without Tying

Platforms compete in prices to attract consumers in both sides of the market. Each intermediary \(i\)’s objective function is given by:

\[
\max_{p_i, q_i} m_i (p_i - c) + n_i (q_i - d),
\]  

(3)

where \(m_A\) and \(n_A\) are jointly determined by equations (1) and (2).

I mainly focus on the symmetric equilibrium in this model. To derive the symmetric equilibrium price for content providers, \(p_A = p_B = p^*\), let me consider a situation where each platform intermediary has market share of \(\frac{1}{2}\) in the consumer side of the market \((n_A = n_B = \frac{1}{2})\) and offers consumers the utility of \(\overline{u} = b m_i - q_i\) (gross of transportation costs). Now consider an intermediary’s profit maximization problem given this utility \(\overline{u}\), maintaining \(\overline{u} = b m_i - q_i\) constant. Then by substituting \(n_i = \frac{1}{2}\) and \(q_i = b m_i - \overline{u}\), intermediary \(i\)’s profit can be written as:

\[
\Pi_i = m_i (p_i - c) + \frac{1}{2} (b m_i - \overline{u} - d)
\]

9 In section IV, I extend the model to allow multi-homing on both sides.
The first order condition with respect to \( p_i \) yields:

\[-F(\frac{1}{2} \pi - p_i) (p_i - c) + F(\frac{1}{2} \pi - p_i) - \frac{1}{2} b F'(\frac{1}{2} \pi - p_i) = 0\]  

The first order condition with respect to \( p_i \) yields:

\[-F(\frac{1}{2} \pi - p_i) (p_i - c) + F(\frac{1}{2} \pi - p_i) - \frac{1}{2} b F'(\frac{1}{2} \pi - p_i) = 0\]  

The (symmetric) equilibrium price for content providers is thus given by

\[p^* = c - \frac{1}{2} b + \frac{1}{\eta},\]  

where \( \eta = \frac{F'(\frac{1}{2} \pi - p^*)}{F(\frac{1}{2} \pi - p^*)} \) can be considered as a measure of the price elasticity of content

supply.

In deriving the symmetric equilibrium price for final consumers, \( q_A = q_B = q^* \), I first note a one-to-one relationship between \( q_i \) and \( n_i \) from equation (2), given the rival intermediary’s price for final consumers \( q_j \), where \( i = A, B, \) and \( j \neq i \).

\[q_i = q_j + t (1 - 2n_i) + b[F(\pi n_i - p^*) - F(\pi(1-n_i) - p^*)] \]

It turns out to be more convenient to treat \( n_i \) as a control variable for intermediary \( i \) in the consumer side of the market.

\[\text{Max } \Pi_i = m_i (p^* - c) + n_i (q_i - d) = F(\pi n_i - p^*) (p^* - c)\]

\[+ n_i \{q_j + t (1 - 2n_i) + b[F(\pi n_i - p^*) - F(\pi(1-n_i) - p^*)] - d \} \]

The first order condition with respect to \( n_i \) yields:

\[\pi F'(\pi n_i - p^*) (p^* - c) + \{q_j + t (1 - 2n_i) + b[F(\pi n_i - p^*) - F(\pi(1-n_i) - p^*)] - d \} - d \]

\[+ n_i \{2t + b \pi [F'(\pi n_i - p^*) + F'(\pi(1-n_i) - p^*)]\} = 0\]
At the symmetric equilibrium, the first order condition (9) is satisfied at \( n_i = 1/2 \). This implies that the symmetric equilibrium price \( q_A = q_B = q^* \) can be characterized as:

\[
q^* = d + t - \pi F \left( \frac{1}{2} \pi - p^* \right) [b + p^* - c]
\]  

(10)

Using equation (6), the symmetric equilibrium price \( q^* \) can be written as

\[
q^* = (d + t) - \pi F - \frac{1}{2} b\pi F',
\]

which can be interpreted as the standard Hotelling price \((d + t)\) adjusted by two terms representing inter-group network externalities in the two-sided markets. The first term \((\pi F)\) represents direct inter-group externalities that an additional consumer bestows upon content providers. The second term \((\frac{1}{2} b\pi F')\) represents the indirect inter-group externalities that an additional consumer gives to other existing consumers through the feedback process in which additional content is provided.

**II.4. Socially Optimal Outcome**

In this subsection, I conduct a welfare analysis in which I derive the socially optimal outcome and compare it with the market equilibrium. Under the Hotelling model with the assumption of a covered market on the consumer side, social welfare depends only on the price to content providers \((p_i)\) that affects the amount of content provided in the market.

With the symmetric outcome, social welfare as a function of the price for content providers \((p_1 = p_2 = p)\) can be written as:

\[
W = 2(p - c) F \left( \frac{1}{2} \pi - p \right) + b F \left( \frac{1}{2} \pi - p \right) + 2 \left[ \int_0^{1/2} \left( \frac{1}{2} \pi - \theta - p \right) dF(\theta) \right] + 2 \left[ \int_0^{1/2} \theta^{1/2} d\theta \right]
\]

(12)
The first order condition with respect to $p$ yields the following socially optimal price for content providers:

$$p^* = c - \frac{1}{2}b$$

(13)

The socially optimal outcome thus requires below-cost pricing for content providers to take into account their positive externalities to consumers.

III. An Analysis of Tying in Two-Sided Markets with Single-Homing Consumers

To analyze the effects of tying on competition in two-sided markets, I assume that intermediary $A$ is also a monopolist in a related market called $M$ with a unit production cost of $c_M$. More specifically, to reflect circumstances in the antitrust case against Microsoft concerning the tie-in of Media Player with the Windows operating system, assume that the good/service $M$ in the monopolized market (operating systems) is necessary for consumers to participate in the two-sided market (streaming multi-media) analyzed in this paper. All final consumers have valuation of $v (> c_M)$ for product $M$. It is assumed that entry into market $M$ is not feasible.\(^{10}\)

I consider the following two-stage game. In the first stage, firm $A$ (the monopolistic supplier of product $M$) decides whether or not to tie the two products. A price game ensues in the second stage with the tying decision in the previous stage taken as given. The timing assumption reflects the fact that the tying decision through product design is a longer term decision that cannot be modified easily compared to the price decision. The outcomes are described below and depend on firm A’s tying decision in the first stage.

\(^{10}\) Firm 1 may have a patent or have an installed base that makes entry unprofitable in the presence of switching costs or network externalities (Farrell and Klemperer, 2001).
III.1. No Tying

If the two products are not bundled, let me assume that consumers buy product M prior to participating in the two-sided market since M is essential for the latter activity. Due to the essentialness of product M, the monopolist can extract consumer surplus from participating in the two-sided market. Let \( \bar{u}^* = b m^* - q^* \) (gross of transportation costs) be the equilibrium utility offered in the two-sided market. Assume further that \((v - c_M)\) is sufficiently large that it is in best interest of firm M to cover the market. The consumer who has the lowest surplus is the one located in the middle of the line whose surplus is given by \( \bar{u}^* - \frac{t}{2} \). The monopolist will charge the price of \( v + \bar{u}^* - \frac{t}{2} \). In the two-sided market I am interested in, the analysis in the previous section applies.

III.2. Tying

Suppose that the monopolist bundles the two products and charges a price of \( \bar{q}_A \) for the bundled product on the consumer side.\(^{11}\) I assume that \( v > c_M \) is sufficiently large that firm A will price the bundled good so that every final consumer purchases it. With \( n_A = 1 \), the number of content providers on platform A is then given by \( F(\pi - p_A) \) when the tying firm charges content providers \( \bar{p}_A \). This implies that the bundled good price is set at a price such that the final consumer located at \( x = 1 \) receives zero surplus:

\[
\bar{q}_A = v + b F(\pi - \bar{p}_A) - t \tag{15}
\]

The tying firm’s profit maximization problem can be written as:

\(^{11}\) Variables corresponding to tying are denoted with a tilde.
\[ \text{Max}_{\tilde{p}_A} \Pi_A = m_A (\tilde{p}_A - c) + n_A (\tilde{q}_A - d) \]

\[ = F(\pi - \tilde{p}_A)(\tilde{p}_A - c) + [v + b F(\pi - \tilde{p}_A) - t - d] \]

(16)

The first order condition with respect to \( \tilde{p}_A \) is given by

\[-F(\pi - \tilde{p}_A)(\tilde{p}_A - c) + F(\pi - \tilde{p}_A) - b F(\pi - \tilde{p}_A) = 0 \quad (17)\]

Thus, the equilibrium bundle price for final consumers (\( \tilde{q}_A^* \)) and the price for content providers under tying (\( \tilde{p}_A^* \)) are characterized as:

\[ \tilde{p}_A^* = c - b + \frac{1}{\eta}, \text{ where } \eta = \frac{F'(\pi - \tilde{p}_A^*)}{F(\pi - \tilde{p}_A^*)} \text{ and } \tilde{q}_A^* = v + b F(\pi - \tilde{p}_A^*) - t \quad (18)\]

**III.3. Welfare Analysis**

In this subsection, I compare the market outcomes under tying and without tying to provide a welfare analysis. There are three channels through which tying can affect social welfare, due to the monopolization of both sides of the market. First, all consumers patronize the tying firm’s platform. This implies that there is less variety in the market. As a result, there are less desirable matches between the consumers and platforms, leading to higher overall “transportation costs.” Second, content is provided only on the tying firm’s platform, whereas the same content is produced on both platforms in the absence of tying. Thus, there are savings in duplication costs under tying. Third, the number of entrants in the content side of the market that determines the availability of content can differ across regimes. The first effect is negative while the second effect is positive. The sign of the third effect is ambiguous. The coordination of consumers on the tying firm’s platform
enhances the incentive to enter the content side of the market. However, the tying firm’s pricing decision in that side of the market can offset this positive effect.

To conduct a more explicit welfare analysis, I will analyze the effect of tying assuming that \( \theta \) is uniformly distributed on \([0,1]\). With the uniform distribution, it can easily be verified from equations (6) and (18) that

\[
p^* = \frac{c}{2} - \frac{b}{4} + \frac{\pi}{4}, \quad \bar{p}^* = \frac{c}{2} - \frac{b}{2} + \frac{\pi}{2}
\]

This implies that the number of content providers under each regime is given by:

\[
m^* = F\left(\frac{1}{2} \pi - p^*\right) = \frac{\pi}{4} + \frac{b}{4} - \frac{c}{2}
\]

\[
\bar{m}^*_A = F(\pi - \bar{p}^*_A) = \frac{\pi}{2} + \frac{b}{2} - \frac{c}{2}
\]

Thus, in the case of the uniform distribution, there is more variety of available content with tying \((\bar{m}^*_A > m^*)\).

To explore the welfare implications of tying, I note that welfare under tying can be written as:

\[
\bar{W} = (\bar{p}^*_A - c) F(\pi - \bar{p}^*_A) + b F(\pi - \bar{p}^*_A) + \left[ \int_{\pi - \bar{p}^*_A}^{\pi} (\pi - \theta - \bar{p}^*_A) dF(\theta) \right] + \left[ \int_0^1 t d\theta \right]
\]

With the uniform distribution, it can be verified that social welfare under each regime are given by:

\[
W = \frac{3}{16} (\pi + b - 2c)^2 - \frac{t}{4}, \quad \bar{W} = \frac{3}{8} (\pi + b - c)^2 - \frac{t}{2}
\]

Thus, the social welfare change due to tying can be written as:

\[
\Delta W = \bar{W} - W = \frac{3}{16} ((\pi + b)^2 - 2c^2) - \frac{t}{4}
\]
The result thus suggests that the welfare implications of tying depend on the relative magnitude of inter-group externalities ($\pi$ and $b$) and the extent of product differentiation. If the extent of inter-group externalities ($\pi$ and $b$) is significant compared to that of product differentiation ($t$), tying can be welfare-enhancing since the benefit from internalizing the inter-group network externalities outweighs the loss of product variety. Otherwise, tying reduces welfare.

**IV. Competition in Two-Sided Markets with Both Sides Multi-Homing**

The analysis above considers situations in which the consumer side of the market is characterized by exclusive intermediation. However, this assumption is at odds with the prevailing condition in many two-sided markets, such as the digital media and the payment card industries. In the digital media case, many users have more than one media player and many content providers offer content in more than one format. The payment card industry portrays a similar picture, with consumers carrying more than one payment card and merchants accepting multiple payment cards. In this section, I modify the basic model to explicitly analyze the possibility of multi-homing on both sides.

**IV.1. Content Providers**

In previous sections, I assumed that there is free entry in the market for content provision and did not specify if the content available across platforms is the same or different. With the assumption of single-homing by consumers, all the consumers’ sole concern is the amount of content available for each platform. However, once multi-homing is allowed on the consumer side, there is a difference; in the symmetric equilibrium with the
same amount of content available for each platform, consumers do not have any incentive to multi-home and the equilibrium identified in section II continues to be an equilibrium even if multi-homing is allowed on the consumer side as long as the same amount – not just the same content – is provided across the platforms. However, if the content is different across the platforms, the previous equilibrium may not survive with the possibility of multi-homing on the consumer side.

In order to make the possibility of multi-homing on the consumer side play a role, assume that there are two types of content available. One type of content is more suitable for one of the two platforms (formats) whereas the other type of content is suitable for both formats. To simplify the analysis, let me assume that when content is of the first type, that is, more suitable for one of the two formats, it is not economically feasible to encode in the other format. More specifically, the total measure of contents potentially available for each format is normalized to 1.12 Among them, the proportion of $\lambda$ is of the first type and thus can be encoded only for a particular format whereas $(1-\lambda)$ can also be encoded in the other format. The existence of exclusive contents available for each format creates incentive for consumers to multi-home. When the second type of content is encoded for both formats, content providers are said to multi-home.

**IV.2. Consumers**

The consumer side of the market is the same as in the previous sections. The only modification is that consumers are now allowed to multi-home. As a result, there are three choices for consumers assuming that the market is covered as before. Consumers can

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12 I assume away fixed cost of creating content in this model since the amount of available content is fixed and the extent of entry by content providers is not an issue.
choose to either single-home or multi-home. If they decide to single-home, they choose one of the two platforms to participate in (See Figure 1).

\[ \lambda - \lambda_{1} \]

Common Contents

Exclusive content for A

Exclusive content for B

Content Provider Side

Platform A

Platform B

Consumer Side

0

1

Single-homing (A)

Multi-homing (A & B)

Single-homing (B)

Figure 1: Two-Sided Markets with Multi-Homing


I am interested in an equilibrium in which both content providers and consumers multi-home. Imagine a situation in which each platform has exclusive contents of measure \( \lambda \) and nonexclusive contents of \((1-\lambda)\) available. In other words, the nonexclusive contents are available for both formats. Consider a consumer located closer to platform A who would thus choose to participate in platform A in a symmetric equilibrium if he chooses only one platform. Now I analyze the consumer’s incentive to multi-home, that is, to participate in platform B in addition to A.
If a consumer located at point $x$ participates in platform $A$, his utility is given by

$$U^A(q_A, x) = b m_A - q_A - tx$$

as before. With the assumption that $m_A = 1$, I have $U^A(q_A, x) = b - q_A - tx$. If the consumer multi-homes, his utility is given by

$$U^{AB}(q_A, q_B, x) = b m - q_A - tx - q_B - t(1-x),$$

where $m$ is the total amount of content available to consumers who multi-home. Since each platform has duplicative contents of $(1-\lambda)$, I have $m = 1 + \lambda$. As a result, the utility from multi-homing is given by

$$U^{AB}(q_A, q_B, x) = b (1 + \lambda) - (q_A + q_B) - t.$$  The location of the consumer who is indifferent between single-homing $A$ and multi-homing is given by:

$$x = 1 - \frac{\lambda b - q_B}{t}$$

Similarly, the location of the consumer who is indifferent toward both single-homing $B$ and multi-homing is given by:

$$y = \frac{\lambda b - q_A}{t}$$

This implies that the number of consumers who single-home platform $i$ is as follows (see Figure 1).

$$n_i = 1 - \frac{\lambda b - q_j}{t}, \text{ where } i = A, B \text{ and } j \neq i. \quad (22)$$

The number of consumers who multi-home is given by

$$n_M = y - x = \frac{2\lambda b - (q_A + q_B)}{t} - 1 \quad (23)$$

Let $N_A$ and $N_B$ denote the total number of consumers who participate in platform $A$ and $B$, respectively. Then, we have
\[ N_A = y = n_A + n_M = \frac{\lambda b - q_A}{t}, \quad N_B = 1 - x = n_B + n_M = \frac{\lambda b - q_B}{t} \]  

\[ x = n_A \quad y = 1 - n_B \]

**Figure 2. The Choice of Consumers**

On the content provider side, the incentives to participate in each platform depend on the configuration of consumers on the other side of the market. Let me assume a situation in which the consumer market is covered and some consumers multi-home, that is, \( N_A + N_B > 1 \) with \( n_M (= N_A + N_B - 1) \) multi-homing consumers. Exclusive content for platform A will be provided if \( \pi N_A - p_A \geq 0 \). For nonexclusive content, the incentives to encode in format A depend on whether the same content is provided for the other format. If it is already provided for format B, the condition for a content provider to multi-home, that is, to encode in duplicative format A, is given by \( m_A - p_A \geq 0 \). With multi-homing on the consumer side, \( N_A > n_A \). This implies that platform A can either charge \( \pi N_A \) and attract only \( \lambda \) exclusive content providers or charge \( m_A \) and attract both exclusive and nonexclusive content providers.

Let me analyze platform A’s profit maximization problem assuming that it serves both exclusive and nonexclusive content providers with nonexclusive content providers
multi-homing. Conditions for such behavior to constitute an equilibrium will be derived later. In such a case, we have  \( n_A = 1 - \frac{\lambda b - q_B}{t} \). Notice that with consumers multi-homing, the number of consumers single-homing A (\( n_A \)) depends only on the other platform’s price charged to consumers (\( q_B \)). Thus, the optimal price for platform A on the content provider side depends on \( q_B \) (under the configuration I consider), and it is given by
\[
p_A^* = m_A = \pi (1 - \frac{\lambda b - q_B}{t})
\] (25)

This implies that under the configuration in which both consumers and content providers multi-home, each platform’s optimal price for each side is independent of the other.

On the consumer side, platform A solves the following problem:

\[
Max_{q_A} (q_A - d) N_A = (q_A - d) \frac{\lambda b - q_A}{t}
\] (26)

Thus, the optimal price on the consumer side is given by
\[
q_i^* = \frac{\lambda b + d}{2}, \ i = A, B
\] (27)

The optimal price on the consumer side implies that \( N_A = N_B = \frac{\lambda b - d}{2t} \). For this to be consistent with consumer side multi-homing, we need \( N_A + N_B > 1 \), that is, \( \lambda b - d > t \), which I assume to hold:

**A1.** \( \lambda b - d > t \)

This condition means that for multi-homing to occur on the consumer side, the amount of exclusive contents and the network benefits for consumers should be sufficiently high compared to the cost and “transportation” parameters.
In addition, for the multi-homing postulated above to constitute an equilibrium, I need to have $\pi_i > \lambda \pi N_i$; that is, attracting both exclusive and nonexclusive content providers yields a higher payoff for platforms than attracting only exclusive content providers. With the equilibrium price of $q_i^* = \frac{\lambda b + d}{2}$ on the consumer side, I have $n_i = \frac{2t - (\lambda b - d)}{2t}$ and $N_i = \frac{\lambda b - d}{2t}$. Thus, the condition can be written as follows:

**A2.** $(1 + \lambda)(\lambda b - d) < 2t$

I assume that both A1 and A2 hold in the remainder of the paper.

**IV.2. Tying**

As in the previous section, I assume that $v (> c_M)$ is sufficiently large that firm $A$ will price the bundled good so that every final consumer purchases it. Given that every consumer has A, I analyze incentives for consumers to multi-home, that is, to participate in platform B in addition to A. Given that all consumers already have A, nonexclusive content providers have less incentive to encode the content in duplicate for format B. I thus consider an equilibrium in which all nonexclusive content is provided only for platform A. For multi-homing to take place under tying on the consumer side, it is necessary that exclusive content for platform B be provided. When there exists $\lambda$ amount of exclusive content for platform B, the *additional* benefit of multi-homing for a consumer located at $x$ from platform B is given by $b\lambda - tx$. This implies that the number of multi-homing consumers is given by $\tilde{n}_M = \tilde{N}_B = \frac{\lambda b - \tilde{q}_B}{t}$, where $\tilde{q}_B$ is the price charged to consumers by platform B. The maximum price platform B can charge to content providers when
\(n_M = \tilde{N}_B\) consumers multi-home is \(\tilde{p}_B^* = \pi \tilde{N}_B\). As a result, platform B’s profit maximization is given by:

\[
\text{Max}_{\tilde{q}_B} \lambda (\pi \tilde{N}_B - c) + (\tilde{q}_B - d) \tilde{N}_B = \lambda (\pi \frac{\lambda b - \tilde{q}_B}{t} - c) + (\tilde{q}_B - d) \frac{\lambda b - \tilde{q}_B}{t} \tag{28}
\]

The first order condition for the above problem yields

\[
\tilde{q}_B^* = \frac{\lambda (b - \pi) + d}{2} \tag{29}
\]

The number of consumers who participate in platform B, and thus multi-home, is given by

\[
\tilde{N}_B^* = \tilde{n}_M^* = \frac{\lambda (b + \pi) - d}{2t} \tag{30}
\]

**IV.3. Welfare Analysis**

To explore the welfare implications of tying, I note that welfare under no tying and tying can be written as:

\[
W = (1 + n_M \lambda) b - (1 + n_M) d - \left[ \int_0^{1-N_B} t x d x + \int_0^{1-N_A} t x d x + n_M t \right] + [\lambda (N_A + N_B) + (1 - \lambda)] \pi - 2 c, \tag{31}
\]

where \(N_A = N_B = \frac{\lambda b - d}{2t}\) and \(n_M = \frac{\lambda b - d}{t} - 1\)

\[
\tilde{W} = (1 + \tilde{n}_M \lambda) b - (1 + \tilde{n}_M) d - \left[ \int_0^{1-\tilde{n}_M} t x d x + \tilde{n}_M t \right] + [\lambda (1 + \tilde{N}_B) + (1 - \lambda)] \pi - (1 + \lambda) c, \tag{32}
\]

where \(\tilde{n}_M = \tilde{N}_B = \frac{\lambda (b + \pi) - d}{2t}\)

Thus, the social welfare change due to tying can be written as:
\[ \Delta W = \tilde{W} - W = (\tilde{n}_M - n_M)\left[\lambda (\pi + b) - d\right] + (1 - \lambda) c \]

\[ - \left\{ [\int_0^{1-\tilde{n}_M} tdx + \tilde{n}_M t] - [\int_0^{1-N_B} tdx + \int_0^{1-N_A} tdx + n_M t] \right\} \]

(33)

Notice that \( \tilde{n}_M - n_M = \frac{\lambda (\pi - b) + d + 2t}{2t} > 0 \) by assumption A2, which implies that tying induces more consumers to multi-home and makes platform-specific exclusive contents available to more consumers, which is also beneficial to content providers. The first term in equation (33) thus represents the net beneficial effects of wider availability of exclusive contents due to tying. There are two channels. First, tying induces all consumers to have access to exclusive contents for platform A. Second, the number of consumers who have access to exclusive contents for platform B also increases. This can be seen from the comparison of \( N_B = \frac{\lambda b - d}{2t} \) to \( \tilde{N}_B = \frac{\lambda (b + \pi) - d}{2t} \). In addition, non-exclusive content providers need to participate in only one platform with tying, rather than both, since every consumer on the other side participates in platform A. The second term represents such cost savings associated with non-duplication of contents.\(^{13}\) Both effects are positive. However, tying may increase overall “transportation costs”, which is represented by the third term.

Nonetheless, the simple structure of the model yields an unambiguous answer concerning the welfare effects of tying. To see this, I manipulate equation (33) as

\[ \Delta W = \tilde{W} - W = (\tilde{n}_M - n_M)\left[\lambda (\pi + b) - d - t\right] + (1 - \lambda) c \]

\[ - \left\{ [\int_0^{1-\tilde{n}_M} tdx] - [\int_0^{1-N_B} tdx + \int_0^{1-N_A} tdx] \right\} \]

(34)

\(^{13}\) In this sense, tying induces more multi-homing on the consumer side but less multi-homing on the content provider side.
The first-term in equation (34) is still positive by assumption A1. In addition, the expression in the curly bracket is negative since

\[ \bar{n}_M = \frac{\lambda(b + \pi) - d}{2t} > \frac{\lambda b - d}{2t} = N_A = N_B. \]

Therefore, \( \Delta W = \bar{W} - W > 0 \); that is, tying is welfare-enhancing in this simple model.

To explore the role of multi-homing in the model, it is instructive to consider the welfare effects of tying in a situation where tying prevents consumers from multi-homing.\(^{14}\) Without multi-homing, all consumers will use the tied product only in the two-sided market. This implies that all contents are provided for format A, and exclusive content for format B will no longer be available. The welfare level under tying in the absence of multi-homing is given by

\[ \bar{W}^{SH} = \pi + b - c - d - \int_0^1 t x d x \]  

(35)

If the marginal costs of serving additional customers are small on both sides (i.e., \( c \approx 0, d \approx 0 \)) as in the case of digital media systems, I can approximate \( \Delta W = \bar{W}^{SH} - W \) as

\[
\Delta W = \bar{W}^{SH} - W \approx -(\pi + b)\lambda \left( \frac{\lambda b - t}{t} \right) + \frac{(\lambda b)^2}{4t} - \frac{t}{2} \\
< -(\pi + b)\lambda \left( \frac{\lambda b - t}{t} \right) + \frac{(\lambda b)^2}{4t} - \frac{t^2}{4t} \\
= -\frac{\lambda b - t}{4t}(4\lambda \pi + 3\lambda b - t) < 0 \]  

(36)

Thus, tying is unambiguously welfare-reducing if multi-homing is not allowed. This result is in sharp contrast to the result obtained with the assumption of multi-homing, and it

\(^{14}\) This would be the case if the monopolist engages in technical tying so that it designs its product in a way such that a competitor’s product cannot interoperate with the tying product. My model suggests that such a practice can be anti-competitive.
highlights the importance of explicitly considering the role of multi-homing in the antitrust analysis of network industries.

V. Concluding Remarks

I analyze the effects of tying arrangements on competition in markets with indirect network effects by using the framework of two-sided markets. In particular, I develop a model of network competition that explicitly incorporates the possibility of multi-homing. I consider two possible cases: one in which only the content side is allowed to multi-home, with the consumer side of the market being characterized by exclusive intermediation, and the other in which both content providers and final consumers are allowed to participate in both platforms.

My analysis is motivated by the prevailing condition in the digital media market in which content providers encode in multiple formats and consumers use multiple media players. Multi-homing has the potential to counteract the tendency toward tipping and the lock-in effects in industries with network effects. As a result, tying does not automatically foreclose competing products. Even in the case where tying leads to the foreclosure of competing products, as in the model with exclusive intermediation on the consumer side, the welfare implications of tying can be subtle and ambiguous. Therefore, we need to be cautious when applying the traditional theory of network effects and tipping to two-sided markets. I conclude by mentioning a couple of avenues along which the current analysis can be extended.
First of all, I have assumed that there is an exogenous amount of exclusive content available for each format.\textsuperscript{15} This can be justified if the two platforms are technically differentiated, and some content is more appropriate for one particular type of technology. However, exclusive content can be endogenously created through the use of exclusive contracts. A recent paper by Doganoglu and Wright (2006b) analyzes the ability of an incumbent to use exclusive contracts to deter entry by a more efficient entrant in a market characterized by network effects. They find that exclusive contracts can be anticompetitive if consumers can join only a single firm. With the possibility of multi-homing, however, they find that contracts that only require consumers to commit to purchase from the incumbent is not anticompetitive, unless they prevent consumers from also buying from the entrant. In my model, an interesting question would be if exclusive contracts can be used by non-tying firms to create incentives for consumers to multi-home when the monopolist ties.\textsuperscript{16}

In addition, many network industries are in dynamic and technology-driven high-tech fields. Despite the central position that innovation occupies in the performance of such industries, the model in this paper has been mainly concerned with pricing implications of tying in network industries. One important extension in my model would be to analyze how the possibility of multi-homing shapes the incentives to innovate in network industries.

\textsuperscript{15} See also Hagiu (forthcoming) who considers the possibility of seller multi-homing \textit{exogenously} given.
\textsuperscript{16} See also Armstrong and Wright (forthcoming) for an analysis of exclusive contracts in two-sided markets where exclusive contracts are used as a device to foreclose the market and undermine the competitive bottleneck equilibrium.
References


